BHARATHIAR UNIVERSITY, COIMBATORE – 641 046

M.Phil. / Ph.D. – MATHEMATICS

(FT/PT Effective from 2008-2009)

Paper III : Special Paper (anyone of the following)

1. Nonlinear Dynamics
2. Abstract Control Theory
3. Advanced Topics in Fluid Dynamics
4. Symmetries of Differential Equations
5. Solid Mechanics
6. Graphs and Networks
7. Optimization Techniques
8. Advanced Functional Analysis
9. Composition Operators
10. Algebraic Topology
11. Fuzzy Sets and Systems

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M.Phil Mathematics FT/PT Effective from 2008 - 2009

Paper III - Special Papers : 1. Nonlinear Dynamics

Unit I: The Dynamics of Differential Equations

Integration of linear second order equations - Integration of nonlinear second order equations - Dynamics in the phase plane - Linear Stability analysis - Non autonomous systems.

Unit II: Hamiltonian Dynamics

Lagrangian formulation of Mechanics - Hamiltonian formulation of Mechanics Canonical transformations - Hamilton-Jacobi equation and action - angle variables - integrable Hamiltonians.

Unit III: Classical Perturbation Theory

Elementary perturbation theory - Canonical perturbation theory - Many degrees of freedom and the problem of small divisors - The Kolmogrov- Arnold-Moser theorem.

Unit IV: Nonlinear Evolution Equations and Solitons

Basic properties of the Kdv equation - The inverse Scattering transforms: Basic principles, KdV equation - Other soliton systems - Hamiltonian structure of integrable systems.

Unit V: Analytic Structure of Dynamical Systems

Ordinary differential equations in the complex domain - Integrable systems of ordinary differential equations - Painleve property of partial differential equations.

Treatment as in:
Chaos and Integrability in Nonlinear Dynamics by M.Tabor,
John Wiley and Sons, New York, 1989.

Unit I	Chapter 1 Sections 1.1 - 1.4,1.6
Unit II	Chapter 2 Sections 2.1 - 2.5
Unit III	Chapter 3 Sections 3.1 - 3.4
Unit IV	Chapter 7 Sections 7.1 -7.6
Unit V	Chapter 8 Sections 8.2 - 8.4

2. Abstract Control Theory

Unit I: Abstract Cauchy Problem

The Homogeneous Initial value problem – The inhomogeneous initial value problem – Regularity of mild solutions for Analytical semigroups.

Unit II: Evolution Equations

Evolution systems – Stable families of Generators – An Evolution system in the Hyperbolic case – Regular solutions in the Hyperbolic case – The inhomogeneous equation in hyperbolic case.

Unit III: Nonlinear Evolution Equations

Lipschitz perturbation of linear evolution equations – Semilinear equations with compact semigroups – Semilinear equations with Analytical semigroups.

Unit IV: Controllability

Controllability and Observability.

Unit V: Stability

Exponential stability – Exponential stabilizability and detectability.

Treatment as in:

(a) **A. Pazy, Semigroups of Linear Operators and Applications to Partial Differential Equations,** Springer-Verlag, New York, 1983.
Unit I : Sections 4.1, 4.2 and 4.3
Unit II : Sections 5.1 to 5.5
Unit III: Sections 6.1 to 6.3

(b) **R.F. Curtain and H. Zwart, Introduction to Infinite Dimensional Linear Systems Theory,** Spinger-Verlag, New York, 1995.
Unit IV: Section 4.1
Unit V : Sections 5.1, 5.2

3. Advanced Topics in Fluid Dynamics

UNIT I:

Steady unidirectional flow - Poiseuille flow - two dimensional flow - Paint-Brush model - unsteady unidirectional flow - Flow with circular stream lines - Flow fields in ' which inertia forces are negligible - Lubrication theory.

Treatment as in: **lntroduction to Fluid Dynamics by G.K.Batchelor**, Cambridge University Press. Relevant topics from the book.

UNIT II:

Thermal boundary layers in laminar flow: Derivation of the energy equation - Temperature increase through adiabatic compression - Stagnation temperature - Theory of similarity in heat transfer - Exact solutions for the problem of temperature distribution in a viscous flow - Boundary layer simplifications.

Treatment as in: **Boundary - Layer Theory by Schlichting**, McGraw-Hill 1979. Relevant topics from chapter 12.

UNIT III:

Equation of motion in rotating co-ordinate system - Potential vorticity - vorticity equation - Ertel's theorem - Non dimensional parameters - Rossby number - Ekman number - Geostrophic flow - Taylor-Proudman theorem - Taylor column.

Treatment as in: **An Introduction to the Mathematical Theory of Geophysical Fluid Dynamics by S.Friedlander**. Relevant topics from chapters I to 4.

UNIT IV:

Magnetohydrodynamics: Electrodynamics of moving media - The electromagnetic effects and the magnetic Reynolds number - Alfven's theorem - The magnetic energy - The mechanical equations - Basic equations for the incompressible MHD - Steady Laminar motion - Hartmann flow.

UNIT V:

Magneto hydrodynamic waves - waves in an infinite fluid of infinite electrical conductivity - Alfven waves - Magnetohydrodynamic waves in a compressible fluid - Magneto acoustic waves - Slow and Fast waves - Stability - Physical concepts - Linear Pinch-Kink - Sausage and Flute types of instability - Method of small oscillations - Jeans criterion for gravitational stability.

Treatment as in:

An Introduction to Magneto Fluid Dynamics" by V.C.A.Ferraro and C.Plumpton, Oxford University, 1996.

Relevant topics from chapters 1, 2, 3 and 5.

4. Symmetries of Differential Equations

UNIT I:

Lie Groups of Transformations and Infinitesimal Transformations: Lie Groups of Transformations-Infinitesimal Transformations.

UNIT II:

Extended Transformations (Prolongations) – Multi - Parameter Lie Groups of Transformations; Lie Algebras.

UNIT III:

Ordinary Differential EZquations: Introduction – Invariance of an Ordinary Differential Equation – First Order ODE's

UNIT IV:

Second Higher Order ODE's – Invariance of ODE's Under Multi – Parameter Groups.

UNIT V: Partial Differential Equations:

Introduction – Invariance of a Partial Differential Equation- Invariance for Scalar PDE's

Treatment as in:

Symmetries and Differential Equations by G.W.Bluman and S.Kumei, Applied Mathematical Sciences Vol. 81, Springer–Verlag, New York, 1989.

UNIT I : Chapters 2 (2.1 - 2.2)

UNIT II : Chapter 2 (2.3 - 2.4)

UNIT III : Chapter 3 (3.1 - 3.2)

UNIT IV : Chapter 3 (3.3 - 3.4)

UNIT V : Chapter 4 (4.1 - 4.2)

Reference Books:

1. **G.W.Bluman and S.C.Anco, Symmetries and Integration Methods for Differential Equations**, Applied Mathematical Sciences Vol. 154, Springer –Verlag, New York, 2002.
2. **P.J.Olver, Applications of Lie Groups to Differential Equations**, 2nd Edition, Springer-Verlag, New York, 1993.

5. Solid Mechanics

Unit-I: Stress Tensor

Stresses, laws of motion, Cauchy's formula, equations of equilibrium, transformation of coordinates, Plane state of stress, principal stresses, Cauchy's stress quadric, shearing stress, Mohr's circle, stress deviation, stress tensor in general coordinates, physical components of a stress tensor in general coordinates, equation of equilibrium in curvilinear coordinates.

Unit-II: Analysis of Strain

Deformation, strain tensor in rectangular Cartesian coordinates, Geometric interpretation of infinitesimal strain, rotation, compatibility of strain components, properties of strain tensor, strain in spherical and cylindrical polar coordinates.

Unit-III: Linear Elasticity

Generalized Hooke's Law, Stress-Strain relationship for an isotropic elastic material, Basic equation of elasticity for homogeneous isotropic bodies, boundary value problem, the problem of equilibrium and the uniqueness of solution of elasticity, Saint Venant's principle.

Unit-IV: Two Dimensional Problems in Elasticity

Plane state of stresses or strain, Airy stress function for two-dimensional problems, Airy stress function in polar coordinates, Representation of two-dimensional Biharmonic functions by Analytic functions of complex variable.

Unit-V: Elasticity and Thermodynamics

Law of thermodynamics, energy equation, strain energy function, conditions of thermodynamic equilibrium, Thermodynamic restrictions on the stress-strain law of an isotropic elastic material, Generalized Hookes law, including the effect of thermal expansion, Thermodynamic functions for isotropic Hookean Materials

Treatment as in
Foundations of Solid Mechanics by Y. C. Fung, Prentice Hall, Inc, NJ, 1965.

References:

1. **L.S. Srinath, Advanced Mechanics of Solids**, Tata McGraw Hill Pvt. Co. Ltd, New Delhi, 1980.
2. **P.D.S. Verma, Theory of Elasticity**, Vikas Publishing House, Pvt. Ltd. New Delhi, 1988.
3. **S.P. Timoshenko and T.N. Goodier**, Theory of Elasticity, McGraw – Hill International Book Co. New York, 1988.
4. **J.D. Achenbach, Wave Propagation in Elastic Solids**, North-Holland Publishing Co., Amsterdam, 1973.

6. Graphs and Networks

Unit I: Connectivity and Networks

K-connected Graphs: 2-Connected graphs - Connectivity of digraphs - K-connected and K-edge connected graphs - Applications of Monger's theorem. Network Flo'w Problems: Maximum network flow - Integral flows - Supplies and Demands.

Unit II: Perfect Graphs

The perfect graph theorem - Chordal graphs revisited - other classes of perfect graphs - Imperfect graphs - the strong perfect graph conjecture.

Unit III: Matroids

Hereditary systems and examples - Properties of matroids - the span function and duality - minors and planar graphs - matroid intersection and matroid union.

Unit IV: Random Graphs

Existence and Expectation - Properties of Almost All Graphs - Threshold Functions - Evolution and properties of Random Graphs - Connectivity, Cliques and Coloring -- martingales.

Treatment as in:
Introduction to Graph Theory by B.W.Douglas, Prentice Hall of India, 1999.

Unit I : Chapter 4 - Sections: 4.2 and 4.3 (Pages 144 - 172).

Unit II : Chapter 8 - Section: 8.1 (Pages 288 - 320).

Unit III : Chapter 8 - Section: 8.2 (Pages 320 - 347).

Unit IV : Chapter 8 - Section: 8.5 (Pages 405 - 429).

Unit V: Decompositions and Labeling

Factorizations and Decompositions of graphs – Labeling of graphs

Treatment as in:
Graphs and Digraphs by G. Chartrand and L. Leshiak,
Chapman and Hall/CRC, 1996.

Chapter 9: Sections 9.2 and 9.3

7. Optimization Techniques

UNIT I: Dynamic Programming

Elements of the DP Model: The Capital Budgeting - More on the Definition of , the state - Examples of DP models and computations - Problem of Dimensionality in Dynamic programming - Solution of Linear programs by Dynamic programming.

UNIT II: Decision Theory and Games

Decisions under Risk - Decision Trees - Decisions Under Uncertainty - Game Theory.

UNIT III: Inventory Models

The ABC Inventory System - Generalized Inventory Models – Deterministic Models – Just-in-Time (JIT) manufacturing system.

UNIT IV: Queuing Models.

Role of Poisson and Exponential Distribution - Processes Birth and Fousson and Death - Queues with Combined Arrival and Departures - Non-Poisson Queues - Queues with Priorities for Service - Random or Series Queues.

UNIT V: Nonlinear Programming.

Unconstrained Extremal Problems - Constrained Extremal Problems - Nonlinear Programming Algorithm - Unconstrained Nonlinear Algorithms - Constrained Nonlinear Algorithms.

Unit I - Chapter – 10, Unit II - Chapter – 12, Unit – III - Chapter – 14,
Unit – IV - Chapter – 15, Unit V - Chapter – 19, Chapter – 20

Treatment as in:

Operations Research - An Introduction (Fifth Edition - 1996) H.A.Taha, Prentice Hall of India (P) Limited, New Delhi, 1996.

Reference Books:

1) **D. Phillips, A. Ravindran, Solberg, Operations Research: Principals and Practice,** JOHN WILEY & SONS (1976).
2) **S.S.Rao, Engineering Optimization,** (3rd Edition, 1996), New Age International (p) Ltd, New Delhi - 110 002.

8. Advanced Functional Analysis

Unit I : The Spaces $L^p(\Omega)$

Definition and Basic Properties - Completeness of $L^p(\Omega)$ - Approximation by Continuous Functions, Separability – Mollifiers. Approximation by Smooth Functions - Precompact Sets in $L^p(\Omega)$ - The Uniform Convexity of $L^p(\Omega)$ - The Normal Dual of $L^p(\Omega)$.

Unit II: The Spaces $W^{m,p}(\Omega)$

Definition and Basic Properties - Duality, The Spaces $W^{-m,p}(\Omega)$ - Approximation by Smooth Functions on Ω - Approximation by Smooth Functions on R^n - Approximation by Functions in $C_0^\infty(\Omega); (m, p')$ - Polar Sets Transformation of Coordinates.

Unit III: Interpolation and Extension Thorems

Geometrical Properties of Domains - Interpolation inequalities *for* Intermediate Derivatives - Interpolation inequalities Involving Compact Subdomains - Extension Theorems.

Unit IV: Imbeddings of $W^{m,p}(\Omega)$

The Imbedding Theorem - Traces *of* Functions in $W^{m,p}(\Omega)$ on the boundary of Ω - $W^{m,p}(\Omega)$ as a Banach Algebra - Counter Examples and Nonimbedding Theorems - Imbedding Theorems for Domains with Cusps - Imbedding Inequalities Involving Weighted Norms.

Unit V: Compact Imbeddings of $W^{m,p}(\Omega)$

The Rellich-Kondrachov Theorem - Two Counter Examples – Unbounded Domains - Compact Imbeddings of $W^{m,p}(\Omega)$ - An Equivalent Norm for $W^{m,p}(\Omega)$ - Unbounded Domains - Decay at Infinity - Unbounded Domains - Compact Imbeddings of $W^{m,p}(\Omega)$ - Hilbert-Schmidt Imbeddings.

Treatment as in:

Sobolev Spaces by R.A. Adams, Academic Press, New York, 1970. (Chapter II to Chapter IV and Pages 22 - 175)

REFERENCES:

1. **J. Aubin, Applied Functional Analysis**, Wiley, New York, 1977.
2. **K. Deimling, Nonlinear Functional Analysis**, Springer-Verlag, New York, 1985.
3. **K. Yosida, Functional Analysis**, Springer-Verlag, Berlin, 1965.

9. Composition Operators

Unit I: Introduction

Definition and Historical background L^p spaces – Functional - Banach spaces of Functions – Locally convex Function Spaces

Composition Operators on L^p spaces:

Definitions, Characterization and Examples - Invertible Composition Operators

UNIT II: Composition Operators on L^p Spaces

Compact Composition operators - Normality of Composition Operators – Weighted Composition Operators

UNIT III: Composition Operators on Functional Banach Spaces

General Characterizations – Composition Operators on spaces $H_P(D), H_P(D_n)$ - Composition Operators on $H_P(D+)$

UNIT IV: Composition Operators on Functional Banach Spaces

Composition Operators on 1 p - spaces.

Composition Operators on the Weighted Locally Convex functions spaces

Introduction. Characterization and Classical Results

Some Applications of Composition of Operators:

Isometries and Composition Operators

UNIT V: Some Applications of Composition Operators

Ergodic Theory and Composition Operators,

Treatment as in:

Composition Operators on Function Spaces by R. K. Singh and J.S. Manhas,
N.H Publications.

Unit I : Sections 1.1 – 1.4, 2.1, 2.2
Unit II : Section 2.3 - 2.5
Unit III : Sections 3.1 – 3.3
Unit IV : Sections 3.4, 4.1, 5.1
Unit V : Section 5.2

Reference Books:

1. **Composition Operators on Hilbert Spaces, Lecture Notes in Mathematics 693, E.A.Nordgren**, Springer, Verlag, New York, (1978).
2. **Ergodic Theory, K.Peterson**, Cambridge University Press, New York, 1983.
3. **Introduction to Linear Operator Theory, V. Istraescu**, Marcel Dekker, Inc., New York, Basel 1981.

10. Algebraic Topology

Unit I: The Fundamental Group

Homotopy of Paths – The Fundamental Group – Covering Spaces – The Fundamental Group of the Circle – Retraction and Fixed Points

Unit II: : The Fundamental Group

The Fundamental Theorem of Algebra – The Borsuk –Ulam Theorem – Deformation Retracts and Homotopy Type – The Fundamental Group of S^n– Fundamental Groups of Some Surfaces

Unit III: Separation Theorem in the Plane

The Jordan Separation Theorem – Invariance of Domain – The Jordan Curve Theorem – Imbedding Graphs in the Plane

Unit IV: The Seifert –van Kampen Theorem

Direct Sums of Abelian Groups **–** Free Products of Groups – Free Groups **–** The Seifert – van Kampen Theorem – The Fundamental Group of a Wedge of Circles

Unit V: Classification of Surfaces

Fundamental Groups of Surfaces – Homology of Surfaces – Cutting and Pasting **–** The Classification Theorem – Constructing Compact Surfaces

Treatment as in:

Topology by J.R.Munkres, Second Edition, Pearson Education, 2006.

Unit I: Chapter 9 (Sec 51-55)
Unit II: Chapter 9 (Sec 56-60)
Unit III: Chapter 10 (Sec 61-64)
Unit IV: Chapter 11 (Sec 67-71)
Unit V: Chapter 12 (Sec 74-78)

References:

1. J.Dugundiji, Topology, Allyn and Bacon, Boston, 1966.
2. W.S.Massey, Algebraic Topology – An Introduction, Springer-Verlag , 1976.

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11. Fuzzy Sets and Systems

UNIT- I

CRISP SETS AND FUZZYSETS: Introduction - Crisp Sets: An Overview - The Notion of Fuzzy Sets - Classical Logic: An Overview- Fuzzy Logic. OPERATIONS ON FUZZY SETS: General Discussion - Fuzzy Complement - Fuzzy Union - Fuzzy Intersection - Combinations of Operations – General Aggregation Operations.

UNIT – II

FUZZY SYSTEMS: General Discussion - Fuzzy Controllers: An Overview - Fuzzy Controllers: An Example - Fuzzy Systems and Neural Networks - Fuzzy Automata - Fuzzy Dynamic Systems. PATTERN RECOGNITION**:** Introduction - Fuzzy Clustering- Fuzzy Pattern Recognition - Fuzzy Image Processing. APPLICATIONS: General Discussion - Natural, Life, and Social Sciences – Engineering – Medicine - Management and Decision Making - Computer Science - Systems Science - Other Applications.

UNIT –III

FUZZY RELATIONS**:** Fuzzy Equivalence Relations - Pattern Classification - Similarity Relations - References. FUZZY GRAPHS**:** Paths and Connectedness - Bridges and Cut Vertices - Forests and Trees - Trees and Cycles - A Characterization of Fuzzy Trees - (Fuzzy) Cut Sets - (Fuzzy) Chords, (Fuzzy) Cotrees, and (Fuzzy) Twigs - (Fuzzy) 1- Chain with Boundary 0, Coboundary, and Cocyles - (Fuzzy) Cycle Set and (Fuzzy) Cocycle Set -. Fuzzy Line Graphs - Fuzzy Interval Graphs - Fuzzy Intersection Graphs - Fuzzy Interval Graphs - The Fulkerson and Gross Characterization - The Gilmore and Hoffman Characterization – Operations on Fuzzy Graphs - Cartesian Product and Composition - Union and Join - On Fuzzy Tree Definition - References. APPLICATIONS OF FUZZY GRAPHS**:** Clusters - Cluster Analysis – Cohesiveness - Slicing in Fuzzy Graphs - Application to Cluster Analysis - Fuzzy Intersection Equations - Existence of Solutions.

UNIT – IV

FUZZY TOPOLOGICAL SPACES**:** Definitions - Concept of a Fuzzy Point and Its Neighborhood Structure - Fuzzy Points and Level Sets - Local Base - A Counter Example - Closure and Kuratowski's Theorem on 14 Sets - Accumulation Points: Generalization of C.T.Yang's Theorem - Ω-Accumulation Points: Lindelof Property - Subspaces. FUZZY PRODUCT INDUCED SPACES**:** Fuzzy Product Spaces - The Functions W and ℑ - Fuzzy Continuity – Product - Induced Spaces. FUZZY NETS AND FUZZY CONVERGENCE: Fuzzy Nets - Fuzzy Upper and Lower Limit - Uniqueness of Convergence Theorem on Iterated Limits - Fuzzy Subnets and Subsequences - A One – to - One Correspondence Between Convergence Classes and Fuzzy Topologies - Fuzzy Continuous Convergence.

UNIT – V

INTUITIONISTIC FUZZY SETS: Definition - Operations, Relations and Properties - IFS of certain level - Cartesian Product and IF Relations - Necessity and Possibility Operations - Topological Operations. OTHER EXTENSIONS OF INTUITIONISTIC FUZZY SETS: Intuitionistic L-Fuzzy Sets - Intuitionistic Fuzzy Sets over Different Universes - Temporal Intuitionistic Fuzzy Sets - Intuitionistic Fuzzy Sets of Second Type - Some Future Extensions of Intuitionistic Fuzzy Sets.

Treatment as in:

UNIT I and UNIT II

1. George J. Klir and Tina A. Folger, ”Fuzzy Sets, Uncertainty and Information”, Prentice Hall of India Private Limited, 2008.

UNIT III

2. John N. Mordeson and Premchand S. Nair, ”Fuzzy Graphs and Fuzzy Hyper graphs”, Physica - Verlag Heidelberg, 2000.

UNIT IV

3. Ying-ming Liu, Mao-kang Luo , “Fuzzy Topology”, World Scientific Pub., 1997.

UNIT V

4. Krassimir T. Atanassor, ”Intutionistic Fuzzy Sets”, Physica - Verlag, 1999

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